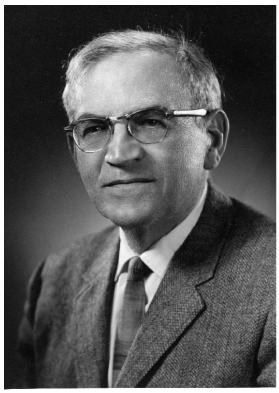
Afterword

It is very difficult, if not well nigh impossible, to give an exact history of the development of any set of ideas. Nonetheless, there are four persons whose contributions "stand out" when considering the history of total positivity. They are I. J. Schoenberg, M. G. Krein, F. R. Gantmacher, and S. Karlin. Of course they did not work in a vacuum and numerous influences are very evident in their research.

It was Schoenberg who initiated the study of the variation diminishing properties of totally positive matrices in 1930 in Schoenberg [1930], and the study of Pólya frequency functions in the late 1940s and early 1950s. Independently, and unaware of Schoenberg's work, Krein was developing the theory of total positivity as it related to ordinary differential equations whose Green's functions are totally positive. Furthermore, in the mid-1930s Krein, together with Gantmacher, proved the spectral properties of totally positive kernels and matrices, and many other properties (see Gantmacher, Krein [1935], Gantmacher [1936], Gantmacher, Krein [1937], and their influential Gantmacher, Krein [1941], which was later reissued as Gantmacher, Krein [1950], and its translations in German in 1960 and in English in 1961 and 2002). These topics are the foundations upon which has been constructed the theory of total positivity. Karlin's role was somewhat different. His books Karlin, Studden [1966] and Karlin [1968], the latter titled *Total Positivity. Volume 1* (but there is no Volume 2), presented many new results and ideas and also synthesized and popularized many of these ideas.

As the reader has hopefully noted, each chapter of this monograph ends with remarks that include bibliographical references and explanations. However I wanted to take this opportunity to write a "few words" in memory of each of these gentlemen.

I. J. (Iso) Schoenberg (1903–1990) was born in Galatz, Romania and



I. J. Schoenberg, 1903–1990

died in Madison, Wisconsin. His family moved to Jassy in 1910, and he studied mathematics at the university there. He spent three years in Germany studying with both Edmund Landau and Issai Schur. Influenced by Schur, he wrote a thesis in Analytic Number Theory. (From Landau he also took Landau's daughter Charlotte (Dolli) as his first wife. He is one of many exemplars of the well-known fact that mathematical talent is an inherited trait: from father to son-in-law.) Schoenberg was interested in the problem of estimating the number of real zeros of a polynomial, and this led him to his work on variation diminishing transformations and Pólya frequency functions and kernels, which are two major topics in the theory of total positivity. A Rockefeller fellowship fortunately brought him to the United States in 1930. He spent time at the University of Chicago, was a Fellow at the newly established Princeton Institute of Advanced Studies, was on the faculty of Colby College from 1936 to 1941, and was then at the University of Pennsylvania from 1941 to 1965. In 1965 Schoenberg moved to the University of Wisconsin and joined the Mathematics Research Center and the Department of Mathematics. He retired in 1973 but remained mathematically active until his death. He contributed to many areas of mathematics, in particular total positivity and splines. For further information, see de Boor [1988] (and especially Schoenberg's autobiographical "A brief account of my life and work" at the beginning of the first volume), Askey, de Boor [1990], MacTutor, and references therein.

Mark Grigorievich Krein (1907–1989) was born in Kiev and died in Odessa. He was a truly eminent and exceedingly prolific mathematician

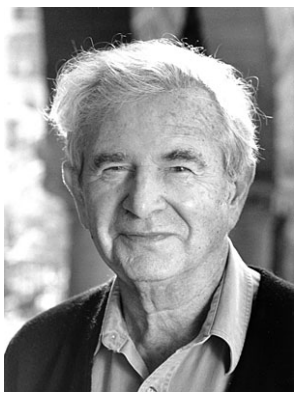
M. G. Krein, 1907–1989

who contributed significantly to and had a tremendous impact upon many different areas of mathematics (see e.g., Gohberg [1989] and Gohberg [1990], MacTutor, and references therein). The story of M. G. Krein, and the mathematical schools he built, is fundamentally marred by the tyranny and antisemitism to which he was constantly exposed. Krein was dismissed from his position at the University of Odessa in 1944 and from his part-time position at the Mathematical Institute of the Ukrainian Academy of Sciences in Kiev in 1952. We can only speculate on what might have been if he had been treated with the respect and dignity that were his due. In 1939 he was elected a corresponding member of the Ukrainian Academy of Sciences. He was never elected a full member. (This prompted the famous mathematical joke. *Ques:* How do you know that the Ukrainian Academy of Sciences is the best academy in the world? *Ans:* Because Krein is only a corresponding member.) From 1944 to 1954 Krein held the chair in theoretical mechanics at the Odessa Naval Engineering Institute, and from 1954 until his retirement he held the chair in theoretical mechanics at the Odessa Civil Engineering Institute. Despite being persecuted Krein received international recognition. He was elected an honorary member of the American Academy of Arts and Sciences in 1968, a foreign member of the National Academy of Sciences (of the USA) in 1979, and in 1982 he was awarded the Wolf Prize. The citation for this prize states: "Krein brought the full force of mathematical analysis to bear on problems of function theory, operator theory, probability and mathematical physics. His contributions led to important developments in the applications of mathematics to different fields ranging from theoretical mechanics to

electrical engineering. His style in mathematics and his personal leadership and integrity have set standards of excellence."

F. R. Gantmacher, 1908–1964

Feliks Ruvimovich Gantmacher (1908–1964) was born in Odessa and he studied there. In 1934 he moved to Moscow, where he resided until his death. Gantmacher is, of course, known for his excellent and influential book *The Theory of Matrices* (Gantmacher [1953]), and his book with Krein; Gantmacher, Krein [1941], [1950]. He was also one of the organizers and editors of the journal *Uspekhi Mat. Nauk* (*Russian Math. Surveys*). Gantmacher was instrumental in the establishment and organization of the well-known Moscow Physico-Technical Institute, where from 1953 until his death he headed the Department of Theoretical Applied Mathematics. An obituary on Gantmacher may be found in Gantmacher [1965].



S. Karlin, 1924–2007

Samuel (Sam) Karlin (1924–2007) was born in Yonova, Poland, but he was raised in Chicago. He earned his PhD from Princeton in 1947 under the guidance of S. Bochner. From 1956 he was a faculty member at Stanford University. The breadth and depth of his interests and contributions in mathematics and in science are astounding. Karlin is the author of more than 450 papers and 10 books, and he had numerous doctoral students. Karlin was passionate about mathematics and science. His passion showed in his lectures, his lifestyle, and his interaction with students and colleagues. Karlin was widely honored, and he received honorary doctorates, numerous prizes (the John von Neumann Theory Prize in 1987, and the National Medal of Science in 1989 to name but two), and he was elected to various academies (see Karlin [2002] and MacTutor).

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